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THE ANALYSIS OF MAGNETOSTATIC WAVES IN A WAVEGUIDE USING THE INTEGRAL EQUATION METHOD

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ABSTRACT

Magnetostatic wave (MSW) propagation in a finite-width ferrite slab placed inside and along a rectangular waveguide is investigated theoretically and numerically. Using the integral equation method, the general solution to the problem of wave propagation has been derived for the first time here in this paper. The thin-slab approximation made the derived solution more tractable and provided the dispersion relations in terms of an infinite determinant. From the presented results, it can be concluded that in order to obtain high values of group time delay over a large bandwidth, thin, narrow slabs placed in the center of the guide must be used. On the other hand, to maximize the device bandwidth, thin slabs placed at the top or bottom of the guide are most appropriate.

INTRODUCTION

Analysis of magnetostatic wave (MSW) propagation in a ferrite material in a normally magnetized structure bound by metal surfaces has been extensively reported in literature.[1-5] MSW propagation in a ferrite slab completely filling a waveguide has also been analyzed and documented.[6] Recently the analysis of magnetostatic waves in a YIG-loaded waveguide was reported.[7,8] The mathematical analysis carried out by these recent investigations was based on a parallel magnetic bias field which led to the propagation of magnetostatic surface waves (MSSW). These waves are highly nonreciprocal with respect to the direction of propagation and unsymmetrical with respect to the slab position in the waveguide. The problem of magnetostatic wave propagation in a YIG slab enclosed in a waveguide with normal magnetic bias field has never been approached and is attempted here for the first time (see Fig. 1).[9,10]

THE INTEGRAL EQUATION METHOD

The analysis of the magnetostatic wave propagation in a finite-width YIG slab appears to be feasible only by the use of the integral equation method.[8]

In this method, an unknown scalar magnetic potential function inside the ferrite slab is assumed to exist. The potential function for all the points inside the slab is denoted by $\Psi(x,y,z)$. Based on $\Psi(x,y,z)$, fictitious magnetic sources can be obtained.

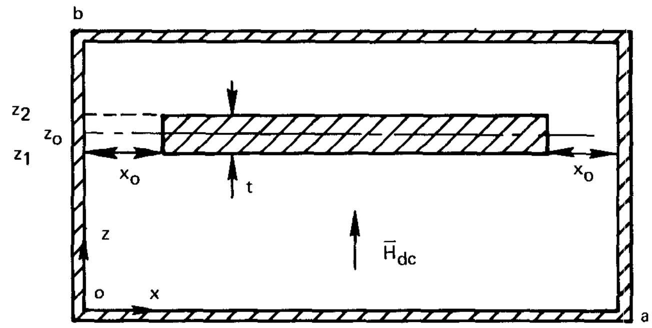

Figure 1. Device configuration for H_{dc} normal to the YIG slab.

Assuming the time dependence to be of the form $e^{j\omega t}$ and the wave propagation to be in the y-direction, the y-variation of all functions would thus be of the form e^{-jKy} where ω is the operating angular frequency and K is the wave number. In this fashion, the scalar magnetic potential function $\Psi(x,y,z)$ inside the YIG region can be written as:

$$\Psi(x,y,z) = \Phi(x,z)\, e^{-jKy} \tag{1}$$

Based on $\Phi(x,z)$ and with the help of proper permeability tensor, fictitious "magnetic sources" can be obtained in terms of $\Phi(x,z)$. Magnetic sources consist of two parts: a) the magnetic volume charge density (ρ_v) and b) the magnetic surface charge density (ρ_s) as follows:

$$\rho_v(x,z) = \frac{\mu-1}{\mu}\Phi_{zz}(x,z) \tag{2}$$

$$\rho_s(x_o,z) = -(\mu-1)\Phi_x(x_o,z) - K_1 K\Phi(x_o,z) \tag{3}$$

$$\rho_s(a-x_o,z) = (\mu-1)\Phi_x(a-x_o,z) + K_1 K\Phi(a-x_o,z) \tag{4}$$

where

$$\mu = 1 + \left[\left(\omega_o\omega_M\right)/\left(\omega_o^2-\omega^2\right)\right]$$

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$$K_1 = \left[(\omega\omega_M)/(\omega_o^2 - \omega^2)\right]$$

$$\omega_o = \mu_o \gamma H_o$$

$$\omega_M = \mu_o \gamma M_o$$

Here μ_o and γ are the free-space permeability constant and gyromagnetic constant (2.8 MHz/Oe), respectively, ω is the operating frequency, and H_o and M_o, in oersteds, are the internal magnetic field and saturation magnetization,[11] respectively (1 Oe = 1000/4 πA/m).

For simplicity of analysis, we assume that the demagnetizing fields are negligible. In this case the external magnetic field (H_{dc}) becomes equal to the internal magnetic field, i.e., $H_{dc} = H_o$.

Considering a uniform waveguide cross section and wave propagation in one unit length, and by means of the magnetic sources and suitable Green's function,[8] an integral expression for the potential function $\tilde{\Phi}(x,z)$ everywhere inside the waveguide (including the ferrite) can be written as:

$$\tilde{\Phi}(x,z) = \iint_{\substack{\text{YIG} \\ \text{AREA}}} \rho_V(x',z')\, G(x,x',z,z')\, dx'dz' + \int_{\substack{\text{YIG} \\ \text{SIDES}}} \rho_S(x',z')\, G(x,x',z,z')\, dz' \qquad (5)$$

Now considering only the points located inside the ferrite slab, from Eq. (5), an integro-differential equation in terms of $\Phi(x,z)$ is obtained:

$$\Phi(x,z) = \int_{x_o}^{a-x_o} \int_{z_1}^{z_2} \frac{\mu-1}{\mu} \Phi_{zz}(x',z')\, G(x,x',z,z')\, dz'dx' - \int_{z_1}^{z_2} \left[(\mu-1)\Phi_x(x_o,z') + K_1 K \Phi(x_o,z')\right] G(x,x_o,z,z')\, dz' + \int_{z_1}^{z_2} \left[(\mu-1)\Phi_x(a-x_o,z') + K_1 K \Phi(a-x_o,z')\right] G(x,a-x_o,z,z')\, dz' \qquad (6)$$

Eq. (6) represents the general formulation to the problem of MSW propagation in a normally magnetized waveguide structure.

COMPUTER SIMULATION AND RESULTS

The integral expression given by Eq. 5 is two dimensional and very difficult to analyze. Assuming the slab to be very thin makes this equation one dimensional and tractable. With this assumption and upon carrying out the z-integrals and evaluating the potential function $\Phi(x,z)$ in Eq. 5 at $z = z_1$, z_o, and z_2, a set of linearly independent equations would be obtained. Recasting these equations into a matrix form yields an infinite coefficient matrix multiplied by an infinite constant vector. Requiring a nontrivial unique solution yields the dispersion relation. This dispersion relation is obtained by setting the determinant of the infinite coefficient matrix to zero. However, for practical purposes, the matrix was properly truncated for best accuracy. Considering only the first-order mode, an extensive computer program was developed to find the determinant of the coefficient matrix. With the aid of the Newton-Raphson root finding technique and along with appropriate computer algorithms, the determinant roots of the dispersion relation were found and plotted.

Fig. 2 shows the effect of slab position in the waveguide on the dispersion characteristics. From this figure it can be seen that the effect of slab position on the dispersion curve becomes pronounced at the higher frequencies in the propagation band. Although the characteristics all converge at the lower end of the propagation band, their slopes are different. This leads to different group time delays as can be seen in Fig. 3. This figure shows the group time delay corresponding to Fig. 2. It can be observed that as the slab is placed toward the center of the guide, the group time delay increases while the propagation bandwidth decreases.

Width effects on the device performance was also studied and the results are shown in Figs. 4 and 5. In Fig. 4, it can be seen that as the normalized air gap increases the propagation bandwidth decreases and the curves flatten out as they shift toward higher frequencies. Fig. 5 shows the corresponding group time delay versus frequency. From this figure, it can be seen that as the slab width decreases (or the air gap increases) the group time delay increases toward higher values with smaller

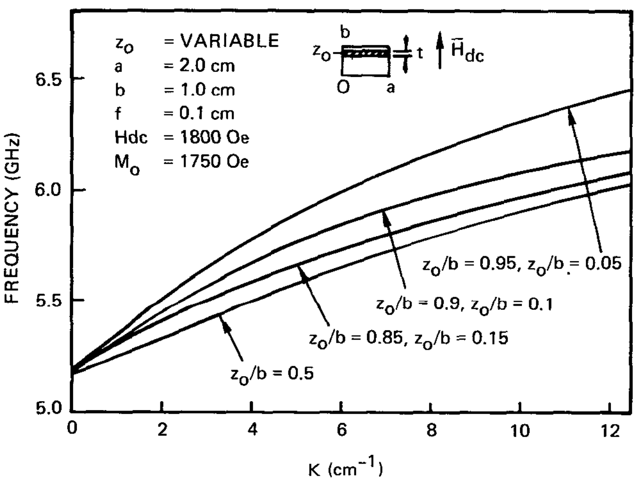

Figure 2. Dispersion curves for different slab positions.

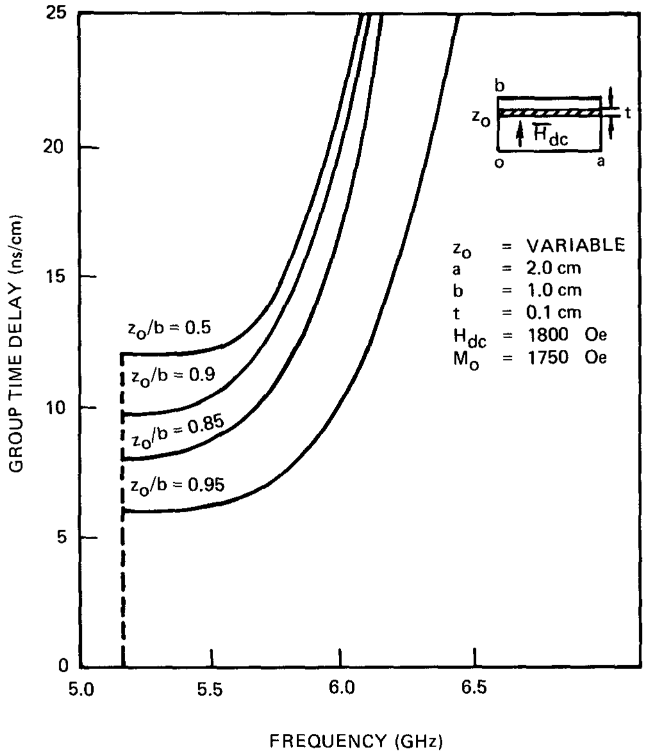

Figure 3. Time delay versus frequency for different slab positions.

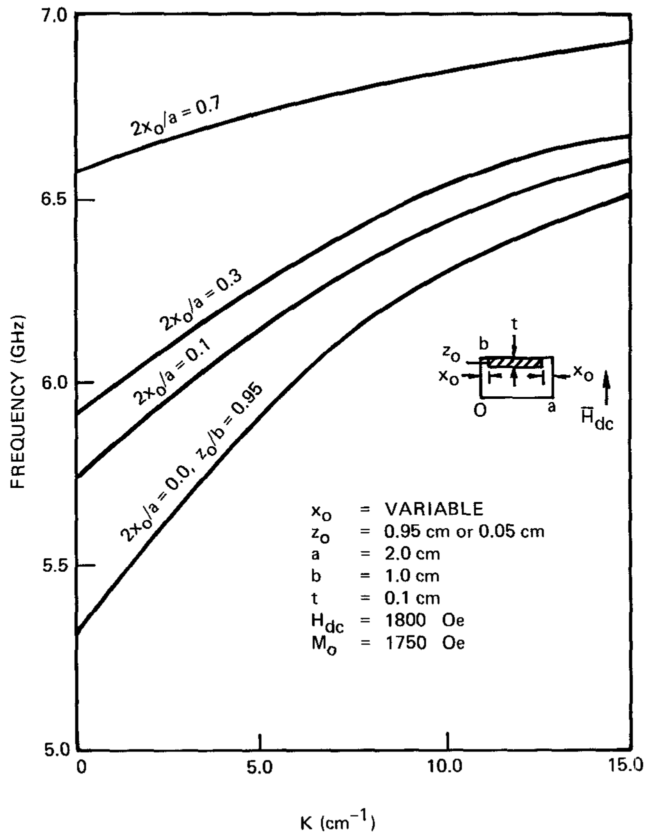

Figure 4. Dispersion curves for different slab widths.

bandwidths as noted earlier. The group time delay at smaller slab widths remains constant in a larger bandwidth and also has a higher value. This property can be used effectively in device design to obtain a constant, high group delay per unit length in a desired frequency band. Fig. 6 plots wave number K versus the normalized air gap ($2x_o/a$). In this figure, the information of Fig. 4 is rearranged in a different fashion. It can be seen that the wave propagation at small slab widths (or large air gaps) is possible only at higher frequencies with smaller wavelengths (or higher K). Once the slab width is chosen, Fig. 6 shows the frequency at which the device must be operated to obtain a certain wavelength, and vice versa.

SUMMARY AND CONCLUSIONS

Magnetostatic wave propagation in a normally magnetized waveguide structure was analyzed and the most general expression embracing the solution to the problem of the MSW propagation in a waveguide, with the use of the integral equation method was derived. Thin-slab approximation led to a set of linearly independent equations which provided the dispersion relations in terms of an infinite determinant. Several important effects were studied with the use of proper truncation procedures. The dependence of the dispersion relations and group time delay per unit length on the position and width of the YIG slab and sample numerical solutions for the first-order mode for several configurations over a frequency range of 5.0 to 7.0 GHz were discussed and the results were presented.

It was also observed that the propagating waves, unlike parallel magnetization case, are reciprocal with respect to the direction of propagation and symmetrical with respect to the slab position in the waveguide.

From these observations, it can be concluded that to obtain high values of group time delay over a large bandwidth, very thin slabs are required. To increase the time delay even more, it is best to choose a narrow width slab and place it in the center of the waveguide.

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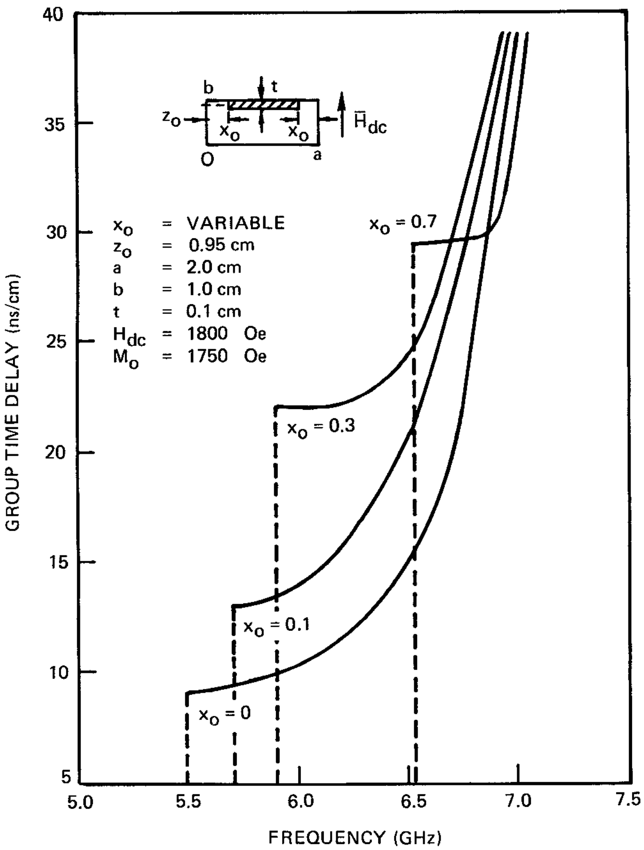

Figure 5. Time delay versus frequency for different slab widths.

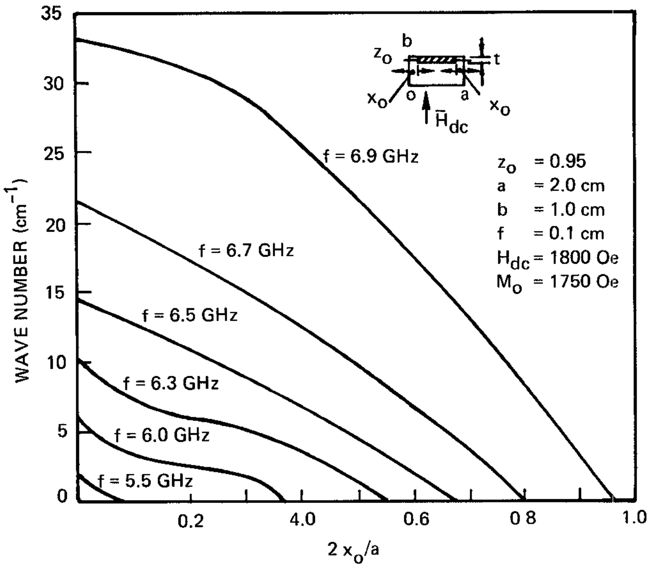

Figure 6. Wave number (k) versus normalized air gap for different frequencies.

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